OBSERVATION OF SHEAR WAVE MODES IN THE ECHOES OF THE LONGITUDINAL WAVE TRANSDUCER

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INTRODUCTION

Longitudinal and shear waves are the fundamental elastic wave modes in an infinite unbounded solid and their velocities are determined by the mass density and elastic moduli of propagationg medium. Thus, elastic moduli could be determined by measuring the phase velocities of longitudinal and shear waves using the corresponding mode transducers.

When an elastic wave is obliquely incident on a solid interface, both of the longitudinal and shear waves are reflected and refracted in accordance with the mode conversion, thus, shear wave velocity can be measured by a longitudinal wave transducer using mode conversion. The velocities of the longitudinal and shear waves were determined by measuring the transit time and ray path length of pulse-echo signals obtained by a longitudinal wave transducer attached on the surface of a cylindrical or spherical specimen[1, 2]. For an incident angle larger than the critical angle of longitudinal wave, only the shear wave is refracted into solid specimen, and the shear wave velocity is determined by using the angle beam transducers[3]. The point-like source of a laser pulse, or a glass capillary breaks generates both longitudinal and shear wave simultaneously, and the velocities of two wave modes can be determined by measuring the transit time and path length of each mode[4].

The directivity pattern of the ultrasonic transducer consists of the main and side lobes. For the normal beam longitudinal wave transducer attached on the surface of a solid, the longitudinal wave is radiated into the direction of the main lobe normal to the transducer surface, and both longitudinal and shear waves are radiated into the direction of the side lobes[5]. Ludwig and his coworker observed the transmission pattern in an aluminum specimen using a normal beam longitudinal wave transducer and compared them with the results of finite element method (FEM)[6]. They could not detect shear wave at the epicenter, but, shear wave was detected at the off-epicenter position.

In the present work, we report small echoes between the large echoes in the pulse-echo signals of the normal beam longitudinal wave transducer. These signals were compared with those of theoretically calculated pulse-echo signals using the Green's function of a point source/point receiver.

THEORETIAL BACKGROUND

For the point source/point detector system, the displacement is given by

$$u_i(\vec{x}, t) = \sum_{j=1}^{3} \int G_{ij}(\vec{x}, \vec{x}'; t - \tau)\, f_j(\tau)\, d\tau, \tag{1}$$

where u is the displacement, $\vec{x}$ is the detector position, G_{ij} is the Green's function, $\vec{x}'$ is the source position, and $\vec{f}$ is the source function. Knopoff[7], Pao and his coworker[8, 9] calculated the displacement due to a step point source in an isotropic medium. They employed to ray theory and Cagniard's method, and considered multiple reflection and mode conversion at the boundaries, considering a point source generates both longitudinal and shear waves which propagate as a spherical wave form, and the mode conversion occurs at the boundary. Hsu developed a computer program to calculate the displacement excited by a step point source in an infinite isotropic plate[10]. These studies showed that a point source generated a longitudinal wave as well as shear wave and one wave mode converts to the other wave mode at the boundary; thus, the displacement is built up by two wave modes.

For a source and receiver of finite area, the effective displacement in the receiver area, $<u_i(t)>$, is given by

$$<u_i(t)>= \frac{1}{S'} \sum_{j=1}^{3} \int_{S'} \int_{S} \int G_{ij}(\vec{x}, \vec{x}'; t - \tau)\, f_j(\vec{x}', \tau)\, d\tau\, d\vec{x}\, d\vec{x}' \tag{2}$$

where S and S' are the area of the source and receiver, respectively.

In the pulse-echo method using a normal beam longitudinal wave transducer, the area of the source and receiver are the same, and the transducer is the source normal to the surface and is sensitive to the normal displacement. Therefore, the output voltage of the transducer can be expressed as

$$V(t) \propto \int_{S} \int_{S} \int G_{33}(\vec{x}, \vec{x}'; t - \tau) f_3(\vec{x}', \tau)\, d\tau\, d\vec{x}\, d\vec{x}'. \tag{3}$$

Taking the transducers to be an uniform piston radiator and the medium to be an isotropic homogeneous infinite plate, the Green's function is dependent on the distance between the source and receiver, $l \equiv |\vec{x} - \vec{x}'|$, and is represented by

$$G_{33}(\vec{x}, \vec{x}'; t) = G_{33}(l; t). \tag{4}$$

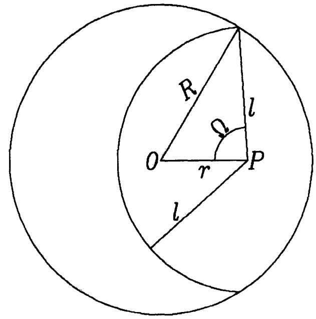

Figure 1. Geometry for calculation of weighting factor.

Substituting Eq.(4) into Eq.(3), the output of receiver is represented as

$$V(t) \propto \int_0^{l_{\max}} \int_{-\infty}^{t} G_{33}(l; t-\tau) W(l)\, f_3(\tau)\, d\tau\, dl \tag{5}$$

where $W(l)$ is weighting factor which is a probability distribution of the distance between source and receiver.

A circular transducer of radius R is shown in Fig. 1. r is the distance between the source point P and the center of the transducer O, the receiving point Q which is l distant from P is on the arc of angle 2Ω which is a function of r and l. The circumference of the possible P is $2\pi r$ and the length of arc is $2\Omega(r,l)\, l$. Therefore, the weighting factor $W(l)$ is given by

$$W(l) \propto \int_0^{R} 2\Omega(r,l)\, l\, 2\pi r\, dr \tag{6}$$

and $\Omega(r,l)$ is given by

$$\Omega(r,l) = \begin{cases} \pi & 0 \le l \le R-r \\ \cos^{-1}\left(\dfrac{r^2+l^2-R^2}{2rl}\right) & R-r \le l \le R+r \\ 0 & l \ge R+r \end{cases} \tag{7}$$

The weighting factor, $W(l)$, is independent on the thickness of specimen, and varies with (l/R) as in Fig. 2.

NUMERICAL CALCULATION

The actual transducer will not be an uniform piston radiator, and the radiation field profile consists of a central plateau surrounded by maximum and minimum rings[11]. These considerations complicate calculation, so in the present calculation, $l_{\max}$ was assumed to be a half of transducer diameter for the simplicity. The weighting factor, $W(l)$, was calculated from Eq. (6), and the Green's functions, G_{33}, as a function of l. The longitudinal and shear wave velocities were taken as the values measured by the pulse-echo overlap method using longitudinal and shear wave transducer, respectively, in the calculation of Green's function. The effective Green's

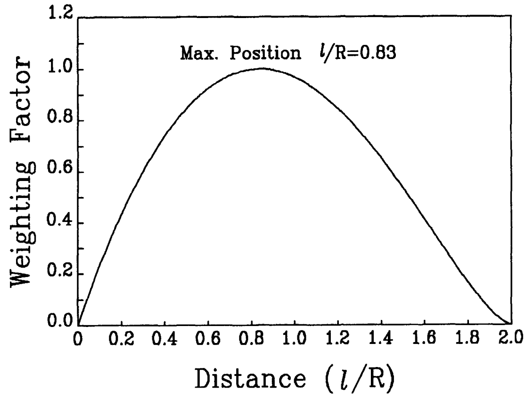

Figure 2. Calculated weighting factor.

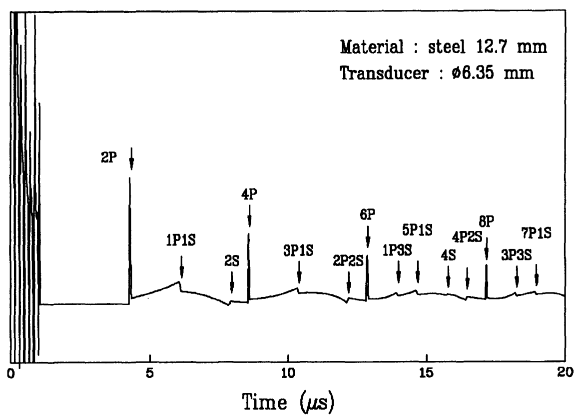

Figure 3. Calculated effective Green's function with considering weighting factor.

function for finite area was obtained by a summation incorporating a weighting factor and Green's function was shown in Fig. 3. The source function was determined by deconvolution of the first back-wall echo.

The predicted pulse-echo signal was obtained by a convolution of the source function and the effective Green's function. Figs. 4 and 5 show the calculated pulse echo signals for steel and aluminum plates of thickness 12.7 mm, respectively. P and denote longitudinal and shear wave modes, respectively and the preceding numbers denote the number of corresponding modes involved in the ray path.

EXPERIMENTAL SETUP

The echo signals of steel and aluminum were observed using the normal bean longitudinal wave transducer of diameter 6.35 mm and center frequency 10 MHz (Ultran WC25-10). The transducer was directly attached to the surface of the specimen using high vaccum grease (Dow Corning Co.). An ultrasonic pulse generator/receiver (Ultran BP9400A) was used to excite the transducer and to receiver

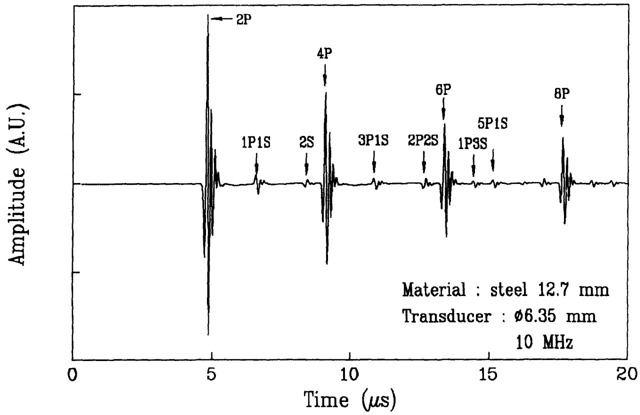

Figure 4. Calculated pulse-echo signal for the steel plate of thickness 12.7 mm.

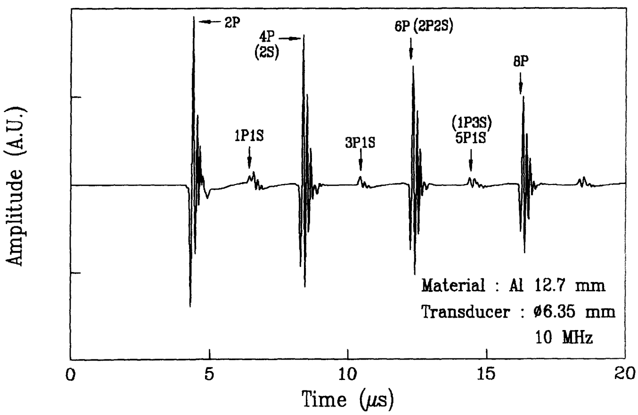

Figure 5. Calculated pulse-echo signal for the aluminum plate of thickness 12.7 mm.

the pulse echo signals. Digital oscilloscope (Lecroy 9410) was used to obtain the pulse-echo signal and to measure the arrival times of each echo.

RESULTS AND DISCUSSION

Pulse Echo Signals and wave speeds

Figs. 6 and 7 show the measured pulse-echo signals for the steel and aluminum plate of thickness 12.7 mm, respectively. The large back-wall echoes correspond to the 2P, 4P and 6P which are pure longitudinal wave mode, however, the small echoes of 1P1S, 2S, 3P1S and 2P2S, between the large back-wall echoes, are the shear wave modes in the ray path.

The agreement between the arrival times of echoes calculated and measured in steel and aluminum plates is good as listed in Table 1 and 2. The 2S echo is a pure shear mode, therefore, the normal beam longitudinal wave transducer generates a few shear wave as well as a longitudinal wave. Considering the transducer as an aggregate

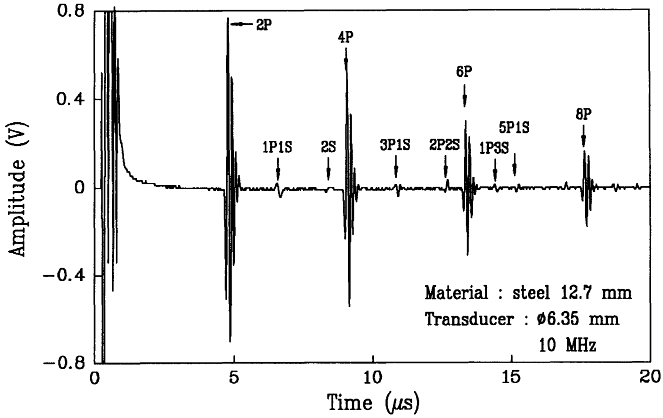

Figure 6. Measured pulse-echo signal for the steel plate of thickness 12.7 mm.

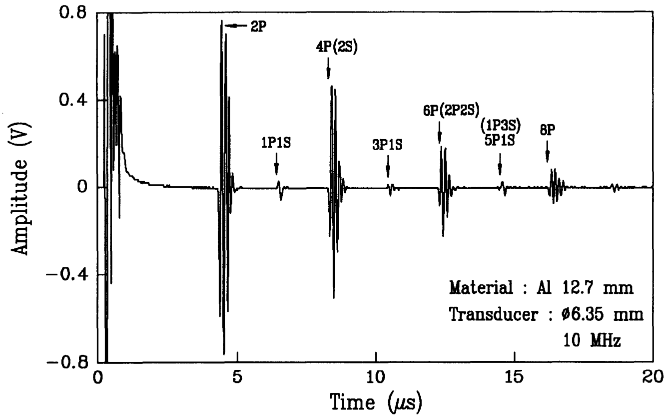

Figure 7. Measured pulse-echo signal for the aluminum plate of thickness 12.7 mm.

of point sources, the one wave mode generated by each point source converts to the other wave mode at the boundary. Since a normal beam longitudinal transducer is used, the amplitude of longitudinal wave generated must be much greater than that of the shear wave. The amplitude of 1P1S echo is larger than that of 2S echo as shown in Fig. 6. This means that the mode conversion of longitudinal wave to shear wave is much stronger than that of shear wave to longitudinal wave. Therefore, the 1P1S echo is the one return as shear wave mode to which the longitudinal wave converts at the boundary.

Since 2P and 4P echo are pure longitudinal wave and 1P1S echo is the shear wave mode in the ray path, the arrival times of 2P, 1P1S and 4P are related to the longitudinal and shear wave velocity as follows.

$$t_{2\mathrm{P}} = \frac{2d}{c_l} \quad t_{1\mathrm{P1S}} = \frac{d}{c_l} + \frac{d}{c_s} \quad t_{4\mathrm{P}} = \frac{4d}{c_l} \tag{8}$$

where d is the thickness of the specimen, and $t_{2\mathrm{P}}$, $t_{1\mathrm{P1S}}$ and $t_{4\mathrm{P}}$ are the arrival times of

Table 1. The arrival times of the echoes in the steel specimen of thickness 12.7 mm (unit=μs).

	2P	1P1S	2S	4P	3P1S	2P2S	6P
measured	4.39	6.15	7.94	8.58	10.52	12.21	12.86
calculated	4.40	6.23	8.08	8.64	10.43	12.23	12.91

Table 2. The arrival times of the echoes in the aluminum specimen of thickness 12.7 mm (unit=μs).

	2P	1P1S	4P	2S	3P1S	6P	2P2S
measured	4.02	6.06	7.98		10.01	11.95	
calculated	4.00	6.09	7.95	8.20	10.04	11.92	12.13

the 2P, 1P1S and 4P echoes, respectively. The longitudinal wave velocity is given by

$$c_l = \frac{2d}{t_{4\mathrm{P}} - t_{2\mathrm{P}}}, \tag{9}$$

and the shear wave velociy by

$$c_s = \frac{c_l d}{c_l t_{1\mathrm{P1S}} - d}. \tag{10}$$

Longitudinal wave velocities in the steel and aluminum specimens obtained by Eq.(9) were 5,927 m/s and 6,400 m/s, respectively. The shear wave velocities in the steel and aluminum plate were 3,247 m/s and 3,085 m/s, respectively, by Eq. (10) and the corresponding shear wave velocities measured by pulse-echo method using the shear mode transducer are 3,232 m/s and 3,126 m/s. The relative differences of the shear wave velocities obtained by two methods are 0.5% for the steel specimen and 1.3% for the aluminum specimen. These results indicates that longitudinal and shear wave velocity can be determined simultaneously from the pulse-echo signals of a normal beam longitudinal wave transducer.

Pao *et al.*[9] and Hsu[10] reported that a point source generates both longitudinal and shear waves and the displacement is built up of two wave modes. The effective Green's function in Fig. 3 is the response to normal impulse input. There exist peaks of the shear mode as well as the peaks of the longitudinal wave mode. The effective Green's function is calculated by considering a transduce of finite area as an aggregate of point sources. The present results are similar to the results for a point source of Pao *et al.*[9] and Hsu[10]. The shear wave modes are clearly brought out in the pulse echo signals calculated by the convolution of the effective Green's function and the source function as shown in Figs. 4 and 6. The pulse echo signals measured using the normal beam longitudinal wave transducer shown in Figs. 5 and 7 agree with the calculated pulse echo signals shown in Figs. 4 and 6. Ludwig and Lord[6] observed the shear wave generated by normal beam longitudinal transducer at the off-epicenter. This means that the shear wave generated by a normal beam longitudinal transducer radiates obliquely to the transducer surface. In the present work, however, the echoes of the shear wave mode were observed in the pulse echo signals of a normal beam longitudinal transducer. Most of the shear wave mode are due to the mode conversion similar to that of the waves generated by the point source and a few shear mode are generated by the transducer.

Other Phenomena Related to the Present Work

The high order echoes of the longitudinal mode(4P, 6P, and 8P) of the pulse echo signal measured in the alunimun specimen in Fig. 7, were distorted by the superposition of 4P on 2S, 6P on 2P2S and 8P on 4S and 6P1S, which was due to the longitudinal wave velocity is close to twice that of shear wave. Therefore, attention is required to measure the wave velocity and attenuation for materials such as aluminum.

As in the present work, longitudinal wave modes were observed in the pulse-echo signal using a shear wave transducer. Those signals are built up in the same way as the echoes of the longitudinal transducer.

CONCLUSION

The pulse echo signals of a normal beam longitudinal wave transducer in steel and aluminum specimens were compared with the calculated pulse echo signals. The large amplitude echoes returning corresponded exactly to the longitudinal wave mode and small amplitude echoes to the shear wave mode. The same phenomena was presented for the normal beam shear wave transducer as well as for the longitudinal wave transducer.

Both the longitudinal and shear wave velocities could be determined from the pulse-echo signal obtained for the longitudinal mode transducer. The velocities of the shear wave determined in this work were compared with those measured by the pulse-echo overlap method, and the relative differences of the shear wave velocities were 0.5% and 1.3% for the steel and aluminum specimens, respectively.

For materials of which longitudinal wave velocity is close to twice that of the shear wave, such as aluminum, the high order echoes of the longitudinal mode overlap with the small echoes, so, the large echoes are distorted.

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